Flow in Conjugate Natural Circulation Loops

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Two natural circulation loops are considered thermally coupled through a heat exchanger common to both. Over the rest of the loops, the heat inflow per unit length is assumed known. A steady-state analysis is made in order to determine the fluid velocities and temperature distributions. Certain general results can be obtained for loops of arbitrary shapes. The special case of square geometries is also considered. The possibility of four different steady-state conditions is demonstrated and the variation of these solutions with respect to the system parameters is discussed.

Nomenclature

c	= specific heat of the fluid
D	= cross-sectional diameter
g	= acceleration due to gravity
$\tilde{g}$	= local component of gravitational acceleration
I	= defined in Eq. (16)
l	= nondimensional total length
L	= total length of loop
l_c	= length of heat exchanger zone
q	= heat inflow per unit length
q_M	= maximum absolute value of q
Q	= nondimensional heat inflow
s	= nondimensional longitudinal coordinate
T	= fluid temperature
T_0	= reference temperature
u	= fluid longitudinal velocity
U	= overall heat transfer coefficient
v	= nondimensional fluid velocity
V	= harmonic mean of v_1 and v_2
x	= longitudinal coordinate
β	= coefficient of thermal expansion
Γ	= Rayleigh number
δ	= nondimensional gravitational component
θ	= nondimensional temperature
ν	= kinematic viscosity of fluid
ρ	= fluid density at reference temperature

Subscripts

1	= first loop
2	= second loop

I. Introduction

PROGRESS has been made in the understanding of natural convection loops of arbitrary geometry with one-dimensional models, a recent review of which appears in Ref. 1. Industrial and other applications, however, make use of variations of such simple systems. One example is the multiple loop system[2,3] in which the fluid moves through interconnected pipes much like an electrical network. Another possibility is the conjugate loop system in which a number of natural circulation loops are physically separate but thermally coupled through heat exchangers. Such types of systems will be discussed in this paper.

Nuclear reactor design makes use of circulation systems based on natural convection, especially with regard to emergency cooling.[4,5] Our interest here is in the conjugate system of the kind described by Grand,[6] considerably simplified in order to permit analysis. Although the discussion in Ref. 6 is directed mainly toward problems associated with a liquid metal fast breeder reactor, many of the considerations apply equally well to light water reactors. The residual heat generated after reactor shutdown is designed to be removed by a normal and a decay heat removal system. In both cases, an intermediate heat exchanger transfers heat from the primary heat-generating loop to a secondary loop. This secondary loop can be either the normal operating system, in which case heat rejection would be to the steam generators, or a special system releasing heat to the atmosphere through an air-heat exchanger. In the present analysis, we will assume that the two loops are operating on natural circulation conditions.

Actual designs can be quite complicated in terms of shape and the number of thermally coupled loops. We will simplify the analysis by considering only two loops with a convective heat exchanger zone. However, fluid properties in the loops can be entirely different.

II. Formulation of the Problem

We consider two closed loops of arbitrary shapes, each of which has constant but different circular cross sections. The working fluids can also be taken to be different. There is a coupling zone through which they exchange heat, but not mass or momentum. Such a conjugate system is shown in Fig. 1. A one-dimensional flow approximation will be made in the governing equations, which have been well discussed in the literature for the single loop.[7] We will deal with steady-state conditions under the Boussinesq approximation and neglect axial heat conduction.

The constituent loops of the system will be referred to by the subscripts 1 and 2. The origin of coordinates is taken at one extreme of the heat exchanger zone, with x_1 and x_2 being the

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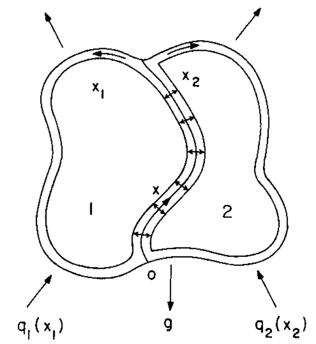

Fig. 1 Arbitrary conjugate loop system.

respective longitudinal coordinates along each of the loops. In the exchanger, the loops are assumed to coincide so that in this interval we can use a common coordinate x. This also means that the local gravity functions for both loops coincide in this zone.

Mass conservation in a loop of constant area and density means that the fluid velocity u_i should be a constant. The momentum equation integrated over the entire loop i is[7]

$$\frac{32\nu_i}{D_i}\,u_i = \frac{\beta_i}{L_i}\int_0^{L_i} \tilde{g}_i(x_i)T_i(x_i)\,\mathrm{d}x_i \tag{1}$$

where ν_i is the kinematic viscosity of the fluid, D_i the diameter, β_i the coefficient of thermal expansion, $\tilde{g}_i(x_i)$ the gravitational acceleration in the negative x_i direction, L_i the total length of the loop, and $T_i(x_i)$ the temperature, all for the loop i. For simplicity, the viscous force in the left-hand side of this equation has been taken to correspond to Poiseuille flow in a straight pipe, but other power-law relations between the friction factor and the Reynolds number may be easily taken into account. The right-hand side of the equation is the total buoyancy force over the whole loop. Integration of the momentum equation over the loop has been used to eliminate the pressure force term.

In order to write the energy equation we will assume that the rate of heat transfer in the exchanger zone is proportional to the local temperature difference between the fluids in the loops; that for the rest of the lengths is prescribed. Thus,

$$\rho_i c_i u_i \frac{\mathrm{d}T_i}{\mathrm{d}x_i} = \begin{cases} \dfrac{4U_i}{D_i}(T_j - T_i) & \text{for} \quad 0 < x_i \le L_c \\[2ex] \dfrac{4q_i}{\pi D_i^2} & \text{for} \quad L_c < x_i \le L_i \end{cases} \tag{2}$$

where ρi is the fluid density, c_i the specific heat, L_c the length of the exchanger section, U_i the overall heat transfer coefficient in the exchanger, and $q_i(x_i)$ the heat inflow per unit length over the rest of the loop. The indices i and j refer to different loops, so that we must take $i = 1$, $j = 2$ to obtain the equation for one loop and $i = 2$, $j = 1$ for the other. By energy conservation in the exchanger, we must have $U_1D_1 = U_2D_2$.

We now have four equations with the same number of unknowns u_1, u_2, $T_1(x_1)$, and $T_2(x_2)$. Nondimensionalization will be carried out using the following variables:

$$\delta_i = \tilde{g}_i(x_i)/g$$

$$s_i = x_i/L_c$$

$$\Delta T_i = q_{iM}/U_iL_c$$

$$\theta_i(x_i) = [T_i(x_i) - T_0]/\Delta T_i$$

$$v_i = u_i\rho_i c_i D_i/4U_iL_c$$

$$l_i = L_i/L_c$$

$$\Gamma_i = \beta_i g \Delta T_i \rho_i c_i D_i^3/128 U_i L_i \nu_i$$

$$Q_i = q_i/\pi U_i \Delta T_i D_i$$

where δ_i, s_i, θ_i, v_i, l_i, and Q_i are the nondimensional gravitational acceleration component, longitudinal coordinate, temperature, velocity, loop total length, and heat inflow, respectively. Γ_i is a modified Rayleigh number that represents the heat flux in each loop. With these variables, Eqs. (1) and (2) become

$$v_i = \Gamma_i \int_0^{l_i} \delta_i(s_i)\theta_i(s_i)\mathrm{d}s_i \tag{3}$$

and

$$v_i \frac{\mathrm{d}\theta_i}{\mathrm{d}s_i} = \begin{cases} \theta_j - \theta_i & \text{for} \quad 0 < s_i < 1 \\ Q_i(s_i) & \text{for} \quad 1 < s_i < l_i \end{cases} \tag{4}$$

in terms of the nondimensional velocity v_i and temperature field $\theta_i(x_i)$. The gravitational function satisfies the geometrical relation

$$\int_0^{l_i} \delta_i(s_i)\,\mathrm{d}s_i = 0 \tag{5}$$

which will be used later. In the heat exchanger section, $\delta_i(x_i)$ is the same for both loops.

The total heat entering the system should obviously be zero for steady-state solutions to exist. That this is reflected in the governing equations is shown by integrating Eq. (4) for both loops and then adding the results. We obtain

$$\int_1^{l_1} Q_1(s_1)\,\mathrm{d}s_1 + \int_1^{l_2} Q_2(s_2)\,\mathrm{d}s_2 = 0 \tag{6}$$

III. General Results

Equations (3) and (4) can be reduced to two transcendental equations in the fluid velocities. To do this, we will first integrate the energy equations to determine the temperature distributions in terms of the velocities and then substitute the results in the momentum equation.

In the exchanger zone ($s_i < 1$), the coordinates s_1 and s_2 are identical and can be replaced by s. Adding Eq. (4) for $i = 1$ and $i = 2$ in this interval and integrating, we get

$$v_1\theta_1(s) + v_2\theta_2(s) = v_1\theta_1(0) + v_2\theta_2(0) \tag{7}$$

where the constant of integration has been determined at $s = 0$. From this, we can obtain one temperature field in terms of the other. Substituting this result in Eq. (4), we get for each loop

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}s} + \frac{\theta_i}{V} = \frac{v_i\theta_i(0) + v_j\theta_j(0)}{v_i v_j} \qquad \text{for} \quad 0 < s < 1 \tag{8}$$

where

$$V = \frac{v_1 v_2}{v_1 + v_2}$$

This can be integrated to give

$$\theta_i(s) = \frac{1}{v_i + v_j}\Big\{v_i\theta_i(0) + v_j\theta_j(0) + v_j[\theta_i(0) - \theta_j(0)]\exp(-s/V)\Big\} \qquad \text{for} \quad 0 < s < 1 \tag{9}$$

The temperatures at the end of the exchanger are $\theta_1(1)$ and $\theta_2(1)$, obtained from Eq. (9). The rest of the energy equation (4) can now be integrated from this point on. We get

$$\theta_i(s_i) = \theta_i(1) + \frac{1}{v_i}\int_1^{s_i} Q_i(s_i')\,\mathrm{d}s_i' \qquad \text{for} \qquad 1 < s_i < l_i \quad (10)$$

Since on completing the loop, the temperature should again be $\theta_i(0)$, we have the following relation:

$$\theta_i(0) = \theta_i(1) + \frac{1}{v_i}\int_1^{l_i} Q_i(s_i')\,\mathrm{d}s_i' \quad (11)$$

From this equation applied to both loops and using Eq. (6), we get

$$\theta_1(0) - \theta_2(0) = \frac{1}{V}\left[1 - \exp(-1/V)\right]^{-1}\int_0^{l_1} Q_1(s_1)\,\mathrm{d}s_1 \quad (12)$$

This relates the temperature of the loops at their origins, showing that they are not independent.

The temperature fields found in Eqs. (9) and (10) can be substituted in Eq. (3) to give

$$v_i = \frac{\Gamma_i v_j}{v_i + v_j}[\theta_i(0) - \theta_j(0)]\left[\int_0^1 \delta(s)\exp(-s/V)\,\mathrm{d}s + \exp(-1/V)\int_1^{l_i}\delta_i(s_i)\,\mathrm{d}s_i\right] + \frac{\Gamma_i}{v_i}\int_1^{l_i}\delta_i(s_i)\int_1^{s_i} Q_i(s_i')\,\mathrm{d}s_i' \quad (13)$$

where we have used condition (5). On substituting Eq. (12) and multiplying by v_i/Γ_i, we have

$$\frac{v_i^2}{\Gamma_i} = [1 - \exp(-1/V)]^{-1}\int_1^{l_i} Q_i(s_i)\,\mathrm{d}s_i \left[\int_0^1 \delta(s)\exp(-s/V)\,\mathrm{d}s + \exp(-1/V)\int_1^{l_i}\delta_i(s_i)\,\mathrm{d}s_i\right] + \int_1^{l_i}\left[\delta_i(s_i)\int_1^{s_i} Q_i(s_i')\,\mathrm{d}s_i'\right]\mathrm{d}s_i \quad (14)$$

We have now obtained the two simultaneous transcendental equations that should be solved for specific heating and gravity functions in order to determine the fluid velocities.

On adding Eq. (14) for the two loops and using Eq. (6), we get

$$I = \frac{v_1^2}{\Gamma_1} + \frac{v_2^2}{\Gamma_2} \quad (15)$$

where

$$I = \int_1^{l_1}\delta_1(s_1)\int_1^{s_1} Q_1(s_1')\,\mathrm{d}s_1'\,\mathrm{d}s_1 + \int_1^{l_2}\delta_2(s_2)\int_1^{s_2} Q_2(s_2')\,\mathrm{d}s_2'\,\mathrm{d}s_2 \quad (16)$$

We have used the fact that

$$\int_1^{l_1}\delta_1(s_1)\,\mathrm{d}s_1 + \int_1^{l_2}\delta_2(s_2)\,\mathrm{d}s_2 = 0 \quad (17)$$

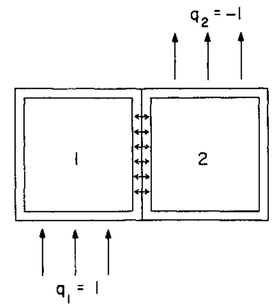

Fig. 2 Thermally coupled square loops.

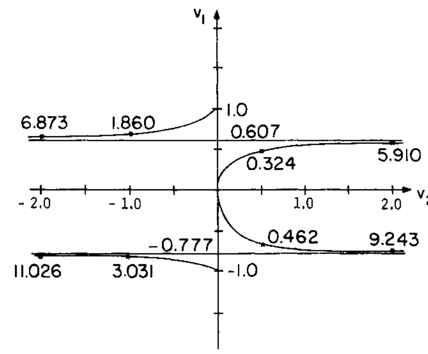

Fig. 3 v_1–v_2 plot for $\Gamma_1 = 1$ and different Γ_2. Γ_2 values are indicated on the curves.

From this, another necessary condition for steady-state solutions to exist is $I > 0$. We have avoided including the equality which, according to Eq. (4), would lead to unbounded temperatures.

On assuming $\int_1^{l_1} Q_1(s_1)\,\mathrm{d}s_1 = 0$ and hence by Eq. (6) $\int_1^{l_2} Q_2(s_2)\,\mathrm{d}s_2 = 0$ also, coupling between the two equations represented by V in Eq. (14) would disappear, and the velocity in each loop would be determined by its own properties and heating functions. There would be no heat transfer along the heat exchanger since, from Eq. (12), we see that $\theta_1(0) = \theta_2(0)$, and then, from Eq. (9), the temperature on either side would be constant and equal.

IV. Specific Example

The general results of the previous section will be applied to the particular case of thermally coupled square loops as shown in Fig. 2. Loop 1 is heated from below and loop 2 is cooled from above, both nondimensional magnitudes being unity. Under these conditions, Eq. (14) becomes

$$\frac{v_1^2}{\Gamma_1} = \left[1 - \exp\left(\frac{v_1 + v_2}{v_1 v_2}\right)\right]^{-1} + \frac{v_1 v_2}{v_1 + v_2} \quad (18a)$$

and

$$\frac{v_2^2}{\Gamma_2} = 1 - \left[1 - \exp\left(\frac{v_1 + v_2}{v_1 v_2}\right)\right]^{-1} - \frac{v_1 v_2}{v_l + v_2} \quad (18b)$$

from which

$$\frac{v_1^2}{\Gamma_1} + \frac{v_2^2}{\Gamma_2} = 1 \quad (19)$$

If the heating and cooling zones had been interchanged, the right-hand side of Eq. (19) would have come out as -1. Consequently, steady-state solutions would be inadmissible under these circumstances without axial conduction.

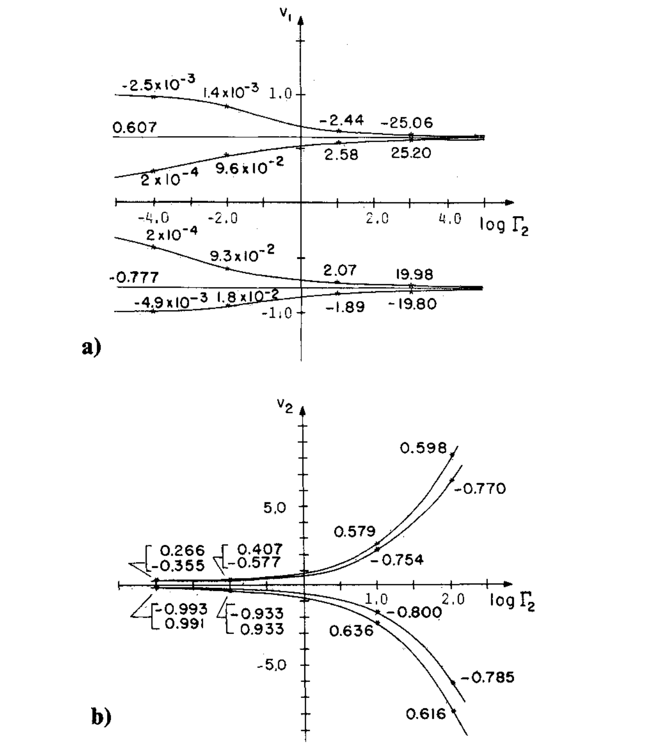

Fig. 4 Variation of a) v_1 and b) v_2 as functions of Γ_2 for $\Gamma_1 = 1$. v_1 and v_2 values are indicated on the curves in a) and b), respectively.

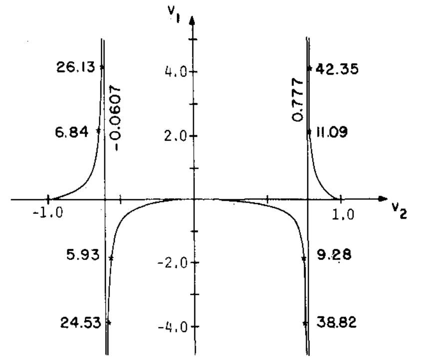

Fig. 5 v_1-v_2 plot for $\Gamma_2 = 1$ and different Γ_1. Γ_1 values are indicated on the curves.

Equations (18a) and (18b) were solved numerically using a Newton-Raphson method. Values of the velocities in a v_1-v_2 plot for fixed Γ_1 and different Γ_2 are shown in Fig. 3. Values of v_1 and v_2 as functions of Γ_2 for $\Gamma_1 = 1$ are shown in Figs. 4a and 4b, respectively. For fixed values of Γ_1 and, Γ_2, there are four distinct solutions for v_1 and v_2, one in each of the quadrants of Fig. 3.

The limits of the velocity v_1 can be easily determined. As v_2 tends to zero, we obtain from Eq. (18a) that

$$v_1^2 \rightarrow \frac{\Gamma_1}{1 - \exp(1/v_2)} \tag{20}$$

From this, and Eq. (18b), we see that

$$v_1 \rightarrow 0, \ \Gamma_2 \rightarrow 0 \quad \text{as} \quad v_2 \rightarrow 0^+ \tag{21a}$$

and

$$v_1 \rightarrow \pm\sqrt{\Gamma_1}, \ \Gamma_2 \rightarrow 0 \quad \text{as} \quad v_2 \rightarrow 0^- \tag{21b}$$

On the other hand, for large values of v_2, we see from Eq. (18a) that

$$v_1^2 = \Gamma_1/[1 - \exp(1/v_1)] + v_1 \quad \text{as} \quad v_2 \rightarrow \pm\infty \tag{22}$$

the solutions to which are $v_1 = 0.60681$ and -0.7772. These are thus the limits between which all the v_1 solutions are forced to lie for arbitrary values of v_2 and Γ_2 as long as $\Gamma_1 = 1$.

Results are shown in Fig. 5 with $\Gamma_2 = 1$ and for different Γ_1. This is similar to Fig. 3 except for the fact that the roles of v_1 and v_2 are reversed.

V. Conclusions

A simplified model of conjugate natural circulation loops has been studied. Axial conduction has been neglected, and the heat fluxes over both loops have been regarded as known except for the heat exchanger region where the heat transfer rate is proportional to the temperature difference. As an additional simplification, frictional forces are considered linear with fluid velocity. Under these conditions, four distinct sets of solutions can be obtained for the fluid velocities, as calculated for a particular example. These four velocities correspond to two in either direction for each loop. Which of these steady-states would occur would depend on initial conditions as well as their stability. If the heating flux of the first loop is fixed, variations of the heating in the other will change its own velocity but maintain the first loop velocity within finite limits.

Real systems should be looked at from the point of view not only of possible two-or three-dimensional motion but also with more general boundary conditions. The time-dependent problem is of interest also, especially regarding the stability of the different steady-state solutions. The present analysis merely outlines the complexity of the problem. It is expected that the simplifying assumptions made here have not altered the basic fact that is the existence of multiple steady-state solutions, but only facilitated their determination.

The simple analysis given in Ref. 6 can reasonably predict a steady-state velocity but not its multiplicity. This is because a temperature distribution is assumed which makes the resulting equation for the velocity, Eq. (1), linear. Multiplicity of steady-state solutions is a consequence of the nonlinear convective term in the energy equation (2). In order to study it, both velocity and temperature have to be considered as unknowns.

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